Skin Effects models for Transmission Line Structures using Generic SPICE Circuit Simulators

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Abstract

A recipe has been developed to model the skin effect for various transmission line structures. The final circuit structure to model this effect can be used with a generic SPICE circuit simulator. The ladder like network[1] element is fairly easy to compute. It uses a minimal increase in CPU time. The model has been verified with experiments.

Introduction

Modern computers rely on faster edge rates. One of the primary concerns of using faster rise time signal propagation is to be able to model all the high frequency effects to be able to correctly predict the behavior of the signal as it propagates through various interconnects. Many of these interconnects appear distributed at the relevant frequencies that makes up the fast rising edge.

This paper addresses one of the aspects of these high frequency effects, namely the skin effect. The basic problem with skin effect is it attenuates the higher frequency components of a signal more than the lower frequency components. This frequency dependent behavior is fairly easy to compute in the frequency domain. However, all digital signals are best treated in the time domain, where the rising edges change shape drastically due to the effect of the skin effect. Typically, the signature of a rising (or falling) edge distortion due to skin effect comprises of a fast rise with a long exponential tail to reach its final value. This is best observed in long cables. However, if the rise time is fast enough, it can show up in smaller interconnect structures also.

Electromagnetic waves do not like to penetrate conductors at high frequencies. At lower frequencies, the field penetrates the entire cross section of the conductor to carry the current. Hence the resistance of the conductor is small and can be computed from equations like $R_{DC} = \rho l/A$. Similarly, in a transmission like structure, at low frequencies, we have to include the internal inductance of both the inner as well as the outer conductor, in addition to the loop inductance in the dielectric. The internal inductance is the inductance arising due to the flux linkage inside the conductor for its own current. This is easy to compute for rectangular or circular conductor. For example, for circular conductor this term is $L_{int} = \mu_0/8\pi = 50$ nH/m. A typical formula for Transmission line structures do not usually include this term, since the derivation for Z_0 or t_{pd} are generally derived for high frequency loss less case. In other words, we assume all the currents are on the surface and the magnetic flux linkages are only in the space between the conductors. For example, a coaxial cable with inner conductor diameter a, and the inner diameter of the outer conductor b, the loop inductance L is given by $L = \frac{\mu_r\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$ and C is given by $C = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$. So for a 50Ω cable, if we have a=1mm and b=2.95mm, we have L=216nH/m and C =97pF/m. Note that L_{int}, in this case, is about 23% of the loop inductance, which we usually ignore.

As the frequency increases, the penetration depth of the electromagnetic field into the conductor decreases. This penetration depth (also known as the skin depth) is given by $\delta = \sqrt{\frac{1}{\pi f\mu\sigma}}$ where f is the frequency, μ, the permeability and σ $(=1/\rho)$, the conductivity of the conductor. (For example, the skin depth of copper is 2μm at 1GHz.) Hence the resistance increases as the effective conductor cross section decreases due to skin effect. Similarly, since the flux inside the conductor decreases, the L_{int} also decreases. The actual frequency dependent R and L expressions can be quite complicated [2,3,4]. For example, in the case of coaxial cables, these contain Bessel Functions and its derivatives. However, the high frequency behavior for a circular cross section conductor (of radius r_0) can easily be approximated as

$$R_{hf} = \omega L_{hf} = \frac{1}{2\pi r_0\sigma\delta}$$

Even though R_{DC} is small, because of skin effect the terms will vary as the square root of frequency ($\sqrt{f}$), and the resistance of the cable may increase by a factor of 200 at 3GHz and the internal inductance L_{int} will drop to very close to zero.

Methodology

The main methodology to model the contributions of skin effect in a lumped circuit simulator like SPICE, lies in the $\sqrt{f}$ dependence of the skin effect impedance. Getting a bandwidth unlimited skin effect model in time domain can be pretty complicated. However, if we decide to use only the generic circuit elements available to us, and we also know the bandwidth of our problem, we can functionally mimic the L and R variation in the desired frequency range.

Intuitively, to mimic the skin effect, a conductor can be thought of as made up of concentric shells. At low frequencies, all the shells are carrying currents, minimizing the resistance and maximizing the internal inductance. As frequency increases, due to the magnetic field inside the conductors, the inner most shells gradually turn off and only the outer shells stay active, thus increasing the resistance and decreasing the internal inductance. This can also be achieved by parallel combination of impedance branches, where each branch will have a resistance and inductance in series.

We can start the first branch with a certain value of R_1 and L_1. As we keep on adding several other parallel branches, the value of the resistance (R_i) of that branch is increased by a factor of x of the value of the resistance of the previous branch (R_{i-1}), where as the value of the inductance (L_i) is decreased by the same factor. We keep doing this depending on the bandwidth of the problem that we are trying to solve. We have found that, by choosing x= $\sqrt{10}$, we can add one additional branch for each decade of frequency. For example, if we need the model to be valid for 3 decades of frequency, then the model only needs to have 3 total branches. The value of the first R and L can be solved fairly easily. We note that the parallel combination of all the (R) resistances should be equal to R_{DC}. Similarly, L can be solved at low frequencies, iteratively by equating the phase of the combination to the

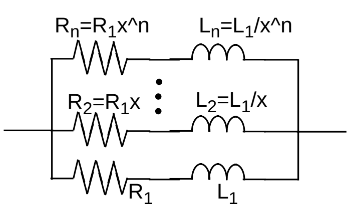

Fig.1: R-L Ladder

low frequency phase of the R_{DC} and L_{int} circuit. The solution of these values are tabulated below:

Table 1: Skin Effect R and L factors

# of Branches	R_1	L_1
1	R_{DC}	L_{int}
2	1.32 R_{DC}	1.68 L_{int}
☛ 3	1.42 R_{DC}	1.94 L_{int}
4	1.45 R_{DC}	2.03 L_{int}
5	1.457 R_{DC}	2.06 L_{int}
6	1.461 R_{DC}	2.07 L_{int}

As an example, consider a circular cable with R_{DC}=5.487mΩ, and the internal inductance, L_{int} =50nH. If we want to construct a three branch skin effect circuit representation, then we can see that in mΩ/m and nH/m, the values are given in the adjacent diagram. Essentially, R_1=1.42 x R_{DC} = 7.771, and L_1=1.94 x L_{int} = 97.1 and x=3.16. In the next branch R is increased by x and L decreased by x. In the third branch it is changed by x^2.

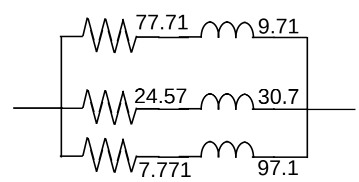

Fig.2: 3 Branch Ladder in mΩ and nH

We have compared the frequency dependent R & L with analytical calculations and Spice simulations. The

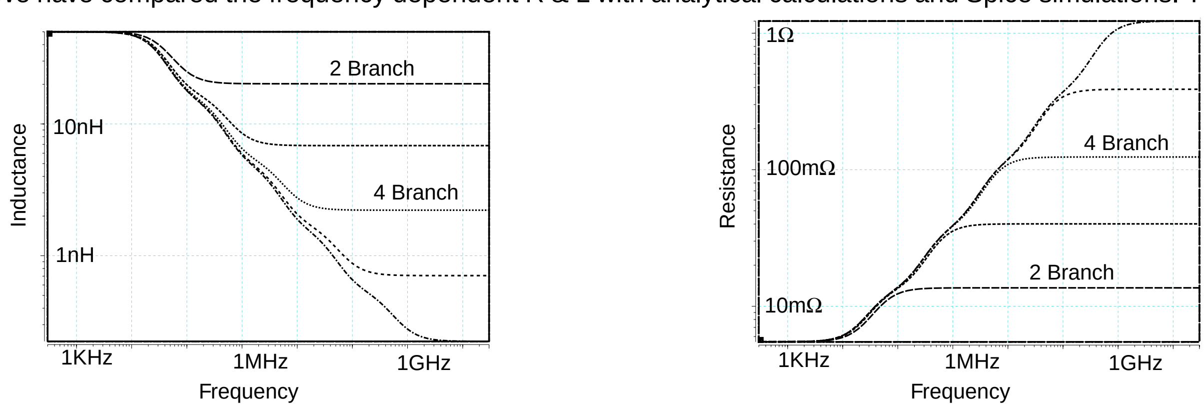

Fig.3a &3b: Inductance & Resistance plot for1-6 ladder circuits

value of R & L of the Spice simulation is shown here. The agreement with theory is pretty good. The bandwidth over which this circuit is valid can be extended by adding additional branches. With 6 branches the model is valid well over 1GHz.

The minimum and maximum frequency range of the circuit can be easily calculated. The minimum frequency f_1 can be defined as the frequency when the resistance is larger by 1% of R_{DC} due to skin effect. Similarly, the maximum frequency f_2 is the frequency of the 3dB point of the L-C filter of the lumped representation of the transmission line. For our case, f_1 is about 30KHz and f_2 is about 30GHz.

Discussion

We have measured the pulse shape distortion of various coaxial cables using TDR. The Spice model as well as the analytical equations agree very well within the bandwidth of the measurement. One of the com

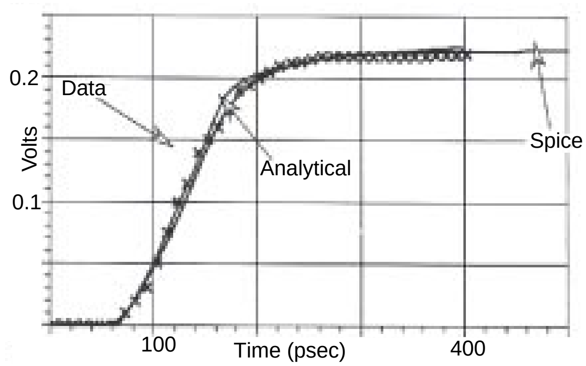

Fig.4: Comparison of the Data, Spice simulation and Analytical results are shown here. The sample is a 5 meter co-axial cable. The Y axis is Voltage and the X axis is time in seconds.

parison is attached above. The errors and the initial slopes are given below.

Table 2: Comparing simulation with the measured Data of a 5m Cable

Technique	Simulation time (sec)	Delay Error (%) @0.12V	Initial Slope (psec) @0.05 to 0.15	Volt Error(%) @0.22V
Spice Recipe	5	0.3	55	2
Analytical[3]	2	0.7	50	5

We have observed that to model coaxial cables, the easiest method is to use a T line model for the cable. 1 lump skin effect model is good enough and fast enough for this comparison. We have found good success when we keep the value of the AC resistance per lump small compared to the characteristic impedance of the transmission line.

We are successful with this scheme to model rectangular conductors also. We have also studied[5] various coaxial cables. Some of these are mixed materials for both the inner and the outer conductors and some of them have hollow inside conductors. Some of the cables also contain magnetic material. The conclusion is that, we can either calculate the spice R and L values for the first branch either from theory or by using spice to fit the experimental data. It turns out that for pure conductors or when the materials are very similar, calculating from theory is easier. If the materials are very different or have hollow conductor, then spice fitting of the experimental data works the fastest, as the theory, though solvable, can get quite complicated. In all cases, after the fitting of the two parameters, the agreement within the bandwidth of measurement is within 5%.

Conclusions

We have defined a methodology to model the skin effect of transmission line structures using generic Spice circuit simulators. The model is fairly easy and intuitive, and computes with minimal increase in CPU time. The model is verified with experimental data and analytical equations. The model also works for hollow coaxial cables and cables with mixed materials.

References

1. C. S. Yen, Z. Fazarinc, and R. L. Wheeler, “Time Domain Skin Effect model for Transient Analysis of Lossy Transmission Lines”, Proceedings of the IEEE, **70**, 750-757, 1982
2. S. A. Schelkunoff, “The Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shields”, Bell Systems Technical Journal, **13**, 532-579, 1934
3. R. E. Matick, “Transmission Lines for Digital and Communication Network”, 194-200

4. Ramo, Whinery and Vanduzer, “Fields and Waves in Communication Electronics”, 286-305
5. Bidyut Sen & Richard Wheeler, “Skin Effect of Mixed material cables”, to be published.